# IMO Sequences and Functional Equations Questions

**Level: Intermediate    Ref No: M04**                                   **Puzz Points: 10**

[Cayley 2004 Q4] The first two terms of a sequence are the numbers 1, 2. From then on, each term is obtained by dividing the previous term by the term before that. Thus the third term is obtained by dividing the second term, 2, by the first term, 1.
(a) Write down the first five terms.
(b) Calculate the fiftieth term.
(c) What happens if other non-zero numbers are chosen for the first two terms, but the rule for calculating the next term remains the same?

Solution: (a) $1, 2, 2, 1, \frac{1}{2}$   (b) 2     (c) If first two terms $a, b$ then sequence is
$a, b, \frac{b}{a}, \frac{1}{a}, \frac{1}{b}, \frac{a}{b}, a, b, \ldots$ which repeats every 6 terms.

**Level: Intermediate    Ref No: M10**                                   **Puzz Points: 15**

[Hamilton 2004 Q4] The first two terms of a sequence are the numbers 1, 2. From then on, each term is obtained by adding 1 to the previous term and then dividing by the term before that. Thus the third term is obtained by adding 1 to the second term and then dividing by the first term.
(a) Write down the first five terms.
(b) Calculate the sixtieth term.
(c) What happens if the other non-zero numbers are chosen for the first two terms, but the rule for calculating the next term remains the same?

Solution: (a) 1, 2, 3, 2, 1   (b)  1    (c)   Let first two terms be $a, b$. Then sequence is
$a, b, \frac{b+1}{a}, \frac{b+1+a}{ab}, \frac{a+1}{b}, a, b$. Since each term is based only on previous two terms, we have proven the sequence repeats every five terms.

**Level: Intermediate    Ref No: M49**                                   **Puzz Points: 20**

[Maclaurin 2007 Q1] The first term c of a sequence is not equal to 1. Each time after the first is equal to "(3 more than the previous term) divided by (1 less than the previous term)".
(a) What values of *c* make the sequence recur forever, in the form *c, c, c, c, c, …*?
(b) Is it possible for any term of the sequence to be equal to 1?

Solution: (a) $c = -1$ or $c = 3$  (b) $\frac{x+3}{x-1} = x$. Then $x + 3 = x - 1$ which clearly has no solutions.

[Hamilton 2010 Q3] The first and second terms of a sequence are added to make the third term. Adjacent odd-numbered terms are added to make the next even-numbered term, for example,

*First term + third term = fourth term*
*And    third term + fifth term = sixth term*

Likewise, adjacent even-numbered terms are added to make the next odd-numbered term, for example,

*Second term + fourth term = fifth term*

Given that the seventh term equals the eighth term, what is the value of the sixth term?


Solution: 0